

# THE 17TH AFFINE ALGEBRAIC GEOMETRY MEETING

## ABSTRACTS OF TALKS

★ 7th March (Thursday)

- Takanori NAGAMINE (Niigata University):

Title: **Factorial rational varieties which admit or fail to admit an elliptic  $\mathbb{G}_m$ -action**

Abstract: Over a field  $k$ , we study rational UFDs of finite transcendence degree  $n$  over  $k$ . We classify such UFDs  $B$  when  $n = 2$ ,  $k$  is algebraically closed, and  $B$  admits a positive  $\mathbb{Z}$ -grading, showing in particular that  $B$  is affine over  $k$ . We also consider the Russell cubic threefold over  $\mathbb{C}$ , and the Asanuma threefolds over a field of positive characteristic, showing that these threefolds admit no elliptic  $\mathbb{G}_m$ -action. Finally, we show that, if  $X$  is an affine  $k$ -variety and  $X \times \mathbb{A}_k^m \cong_k \mathbb{A}_k^{n+m}$  then  $X \cong_k \mathbb{A}_k^n$  if and only if  $X$  admits an elliptic  $\mathbb{G}_m$ -action. This is a joint work with Gene Freudenburg (Western Michigan University).

- Kayo MASUDA (Kwansei Gakuin University):

Title: **Hypersurfaces with  $\mathbb{G}_a$ -actions**

Abstract: We give a condition for a factorial affine variety with a  $\mathbb{G}_a$ -action to be described as a hypersurface in ambient space.

- Ryuji TANIMOTO (Shizuoka University):

Title: **Exponential matrices of Heisenberg groups in positive characteristic**

Abstract: We are concerned with exponential matrices, which are polynomial matrices with exponential properties. It is a difficult problem to classify exponential matrices in positive characteristic. We talk about classifying exponential matrices of Heisenberg groups in positive characteristic.

- Kentaro MITSUI (Kobe University):

Title: **Models of torsors under algebraic groups**

Abstract: For a given quasi-projective flat group scheme  $G$  over a normal integral scheme with generic fiber  $G_\eta$ , we study models of étale torsors under  $G_\eta$  with action of  $G$ , especially, their classification and quasi-sections. In the case where  $G$  is the Néron model of an elliptic curve  $G_\eta$  over an excellent integral Dedekind scheme, we show that the action of  $G_\eta$  on a torsor  $X_\eta$  uniquely extends to an action of  $G$  on the minimal regular model of  $X_\eta$ .

- Taro HAYASHI (Kinki University):

Title: **Finite abelian groups of K3 surfaces with smooth quotients**

Abstract: Finite groups which act faithfully on  $K3$  surfaces are well studied. I will talk about my results of finite groups acting faithfully on  $K3$  surfaces such that the quotient spaces are smooth. Especially, I will focus on explaining how to determine finite groups of  $K3$  surfaces with smooth quotients from pairs of smooth rational surfaces and its effective divisors.

★ 8th March (Friday)

- Riku KUDO (Waseda University):

Title: **Generalized Zariski cancellation problem and bundles over pre-varieties**

Abstract: Generalized Zariski cancellation problem asks whether or not  $V \times \mathbb{A}^1 \simeq W \times \mathbb{A}^1$  implies  $V \simeq W$  for varieties  $V$  and  $W$ , but some counter-examples for this problem have been given as principal  $\mathbb{G}_a$ -bundles over pre-varieties. In this talk I would like to introduce the following result; Let  $V$  and  $W$  be  $\mathbb{A}^1$ -bundles over pre-varieties  $X$  and  $Y$ , respectively. If  $Y$  has a dominant morphism to the same dimensional variety with non-negative logarithmic Kodaira dimension, then  $V \times \mathbb{A}^1 \simeq W \times \mathbb{A}^1$  implies  $X \simeq Y$ .

- Adrien DUBOULOZ (Université de Bourgogne):

Title: **Torus actions on normal affine varieties**

Abstract: By a theory developed by Altmann and Hausen, effective actions of tori on normal affine varieties defined over an algebraically closed field of characteristic zero can be fully described by pairs geometrico-combinatorial data consisting of appropriate rational quotients of for the given torus action and certain Weil divisors on whose coefficients are rational polyhedra in the lattice of one-parameter subgroups of the torus. In this talk, I will explain the main steps of a general program aiming at extending this type of description to all normal affine varieties defined over arbitrary base fields and endowed with effective actions of non necessarily split tori. To give a hint of the main ideas of the construction, which rely on an effective description of the interplay between convex geometry and continuous Galois descent, I will mainly focus on a very elementary toy case: normal real affine varieties endowed with effective actions of the circle, the unique non trivial real form of the split torus. (Joint work in progress with R Terpereau (University of Burgundy) and A. Liendo (University of Talca)).

- Masayoshi MIYANISHI (Kwansei Gakuin University):

Title: **Triviality of affine space fibration**

Abstract: Given an affine space fibration  $f : X \rightarrow Y$ , the generic fiber  $X_\eta := X \times_Y \text{Spec} k(Y)$  is a  $k(Y)$ -form of the affine space by the generic equivalence theorem [1]. We consider the triviality of  $X_\eta$  when the fiber dimension is greater than two and the fibration has a relative action of the additive group  $\mathbb{G}_a$  or the multiplicative group  $\mathbb{G}_m$ . With an additional assumption that a given  $\mathbb{G}_a$ -action is  $q$ -tight (to be defined in the text), we give two new proofs of Kaliman's theorem [2], one proof modulo one of two conjectures stated in the introduction and the other by studying singular fibers of  $\mathbb{A}^1$ -fibrations. We also give a theorem on relative linealization of  $\mathbb{G}_m$ -action on  $\mathbb{A}^4$ .

## REFERENCES

- [1] H. Kraft and P. Russell, *Families of group actions, generic isotriviality and linearization*, Transform. Groups **19** (2014), no. 3, 779–792.
- [2] Sh. Kaliman, *Proper  $\mathbb{G}_a$ -actions on  $\mathbb{C}^4$  preserving a coordinate*, Algebra Number Theory **12** (2018), no. 2, 227–258.

- Masatomo SAWAHARA (Saitama University):

**Title: Cylinders in weak del Pezzo fibrations**

Abstract: The del Pezzo fibrations play an important role in both of birational and affine geometry, for instance, it is known that a given del Pezzo fibration  $\pi : V \rightarrow W$  of degree  $d$  contains a vertical cylinder if and only if  $d \geq 5$  and the generic fiber  $V_\eta$  admits a rational point. Instead of del Pezzo fibrations, we observe vertical cylinders found in a weak del Pezzo fibration, which is by definition a projective morphism  $\pi : V \rightarrow W$  of relative dimension two such that the generic fiber  $V_\eta$  is a weak del Pezzo surface that is minimal over the field  $k = \mathbb{C}(W) = \mathbb{C}(\eta)$ . The existence of vertical cylinders in such a weak del Pezzo fibration being translated into that of an  $\mathbb{A}_k^1$ -cylinder in  $V_\eta$ , the essence of problem lies in asking "Which  $k$ -minimal weak del Pezzo surfaces admit  $\mathbb{A}_k^1$ -cylinders?" As a result, we are aware that very few  $k$ -minimal weak del Pezzo surfaces contain  $\mathbb{A}_k^1$ -cylinders, more precisely, a  $k$ -minimal weak del Pezzo surface  $S$  of degree  $2 \leq d \leq 8$  contains an  $\mathbb{A}_k^1$ -cylinder if and only if  $d = 8$ .

- Jooneyong WON (Institute for Basic Science):

**Title: Alpha invariant of weighted Fano threefolds under certain rigidities**

Abstract: Considering birationally superrigidity, rigidity and bi-rigidity, we study alpha invariant explicitly from some motivations,  $K$ -stability, rigidity of direct product, and affine superrigidity.

★ 9th March (Saturday)

- Taro SANO (Kobe University):

**Title: On quasi-smooth weighted complete intersections**

Abstract: Weighted complete intersections (WCI) provide many examples to test conjectures. Iano-Fletcher gave a quasi-smoothness criterion for WCI in low codimension cases. I will explain its generalization to arbitrary codimension and apply it to the effective non-vanishing conjecture for WCI.

- Florin AMBRO (Institute of Mathematics "Simion Stoilow" of the Romanian Academy):

**Title: Seshadri successive minima**

Abstract: The Seshadri constant of a polarized variety  $(X, L)$  at a point  $x$  measures how positive is the polarization  $L$  at  $x$ . If  $x$  is very general, the Seshadri constant does not depend on  $x$ , and captures global information on  $X$ . Inspired by ideas from the Geometry of Numbers, we introduce in this talk successive Seshadri minima, such that the last one is the Seshadri constant at a point, and the first one is the width of the polarization at the point.

Assuming the point is very general, we obtain two results: a) the product of the successive Seshadri minima is proportional to the volume of the polarization; b) if  $X$  is toric, the  $i$ -th successive Seshadri constant is proportional to the  $i$ -th successive minima of a suitable 0-symmetric convex body. Based on joint work with Atsushi Ito.

- Masaru NAGAOKA (University of Tokyo):

**Title: On compactifications of contractible affine threefolds into del Pezzo fibrations**

Abstract: By the contribution of M. Furushima, N. Nakayama, Th. Peternell and M. Schneider, it is completed to classify all projective compactifications of the affine 3-space  $\mathbb{A}^3$  with Picard number one. After that, T. Kishimoto observed that their arguments make use of only the contractibility of  $\mathbb{A}^3$  and that the ambient space are Fano manifolds. In this talk, I will consider compactifications of contractible affine threefolds into another special manifolds, i.e. del Pezzo fibrations. Mainly I will introduce a certain type of such compactifications, which seems to have connection with vertical cylinder, and give you examples of such certain compactifications.

- Tomoaki OHTA (Kyushu Sangyo University):

**Title: Compactifications of  $\mathbb{A}_{\mathbb{C}}^3$  and Abhyankar-Sathaye Embedding Conjecture**

Abstract: In this talk, I will give the affirmative answer for a special case of Abhyankar-Sathaye Embedding Conjecture for closed embeddings of  $\mathbb{A}^3$  into  $\mathbb{A}^4$ , where the ground field is  $\mathbb{C}$ . Indeed, I will determine all the compactifications  $(X, Y)$  of  $\mathbb{A}^3$  such that  $X$  are cubic hypersurfaces in  $\mathbb{P}^4$  and  $Y$  are hyperplane sections of  $X$ . Moreover, for each  $(X, Y)$ , I will construct an automorphism of  $\mathbb{A}^4$  which transforms the hypersurface  $X \setminus Y \cong \mathbb{A}^3$  onto a coordinate hyperplane.

- Karol PALKA (Polish Academy of Science):

**Title: On the structure of open surfaces of negative Kodaira dimension**

Abstract: Let  $S$  be a smooth open surface of negative Kodaira dimension defined over the complex numbers. If  $S$  has a fibration with general fiber  $\mathbb{P}^1$ ,  $\mathbb{C}^1$  or  $\mathbb{C}^*$  then the structure of  $S$  can be studied easily. If no such fibration exists then it is known that  $S$  is necessarily the smooth locus of some log terminal rational surface with more than one singular point. Here much less is known. We will show some conjectures concerning those surfaces and a way to approach them.

★ 10th March (Sunday)

- Shouya YASUDA (Tokyo Metropolitan University):

**Title: New co-tame automorphisms of a polynomial ring**

Abstract: In this talk, we discuss subgroups of the automorphism group of the polynomial ring in  $n$  variables over a field. An automorphism  $F$  is said to be *co-tame* if the subgroup generated by  $F$  and affine automorphisms contains the tame subgroup. In 2017, Edo-Lewis

gave a sufficient condition for co-tameness of automorphisms. Let  $EL(n)$  be the set of all automorphisms satisfying Edo-Lewis's condition. Then, for a certain topology on the automorphism group of the polynomial ring, any element of the closure of  $EL(n)$  is co-tame. Moreover, all the co-tame automorphisms previously known belong to the closure of  $EL(n)$ . In this talk, we give the first example of co-tame automorphisms in three variables which do not belong to the closure of  $EL(3)$ .

- Shigeru KURODA (Tokyo Metropolitan University):

Title: **TBA**

Abstract: