

THE 15TH AFFINE ALGEBRAIC GEOMETRY MEETING

ABSTRACTS OF TALKS

★ 2nd March (Thursday)

- Hideo KOJIMA:

Title: **Normal del Pezzo surfaces of rank one with log canonical singular points**

Abstract: In this talk, we will consider normal complex projective surfaces with rational log canonical singularities and rank one. I will give some partial classification results of such surfaces.

- Taro SANO:

Title: **Deformations of cones over $K3$ surfaces**

Abstract: Deformations of affine cones over projective varieties have been studied by several people. In particular, Pinkham proved that a cone over an elliptic curve is smoothable if and only if it is contained in a smooth del Pezzo surface. In this talk, I will talk about an analogue of this result for cones over $K3$ surfaces.

- Eiichi SATO:

Title: **On surjective morphisms between Fano manifolds of Picard number one**

Abstract: For two $n (> 2)$ -dimensional smooth Fano varieties X, Y with the Picard number one, we consider the properties of a surjective morphism $f : X \rightarrow Y$ by investigating the behavior of a minimal rational curve C in Y which covers Y and the inverse image $f^{-1}(C)$ in X . Particularly when the source space X is a hyperspace in \mathbb{P}^{n+1} , we study under what condition the above morphism f is an isomorphism. Moreover in case that X is del pezzo variety we state the structure of the target space Y .

- Natsuo SAITO:

Title: **Equisingular families of rational double points in characteristic 2**

Abstract: We consider versal deformations of rational double points in positive characteristic. In characteristic 0, it is known that equisingular families of RDPs are trivial; in a versal family of an each RDP (X_0, x) , a fiber other than the central one cannot have the singularity which is isomorphic to (X_0, x) . However, this fails in positive characteristic. We investigate equisingular families of rational double points in characteristic 2.

- Takeshi TAKAHASHI:

Title: **On the number of Galois Weierstraß points whose semigroups are generated by two elements**

Abstract: This is a joint work with Jiryō Komeda. For a nonsingular plane curve, Yoshihara introduced the notion of Galois points. And Yoshihara and others studied the number of Galois points and its distributions. In this talk, we characterize Galois points with a view point of Galois Weierstraß point. And we study the number of Galois Weierstraß points as a generalization of the study on the number of Galois points.

★ 3rd March (Friday)

- Ryūji TANIMOTO:

Title: **A note on the Weitzenböck problem in dimension four**

Abstract: Let \mathbb{k} be an algebraically closed field of positive characteristic p and let \mathbb{G}_a be the additive group of \mathbb{k} . The Weitzenböck problem asks whether the ring of invariants of \mathbb{G}_a acting linearly on a polynomial ring in n variables over \mathbb{k} is finitely generated as a \mathbb{k} -algebra. In this talk, assuming $p = 2$ and $n = 4$, we give a positive answer to the problem.

- Hirokazu NASU:

Title: **Obstructions to deforming curves lying on a $K3$ surface in a Fano 3-fold**

Abstract: We study the deformations of a smooth curve C on a smooth Fano threefold V , assuming that C is contained in a smooth $K3$ surface $S \subset V$. We give a sufficient condition for C to be (un)obstructed in V , in terms of (-2) -curves and elliptic curves on S . Applying this result, we prove that the Hilbert scheme of smooth connected curves on a smooth quartic threefold $V_4 \subset \mathbb{P}^4$ contains a generically non-reduced irreducible component.

- Isac HEDÉN:

Title: **On the Makar-Limanov invariant of certain affine hypersurfaces**

Abstract: A regular function on an affine variety is called an absolute constant if it is invariant with respect to every \mathbb{G}_a -action on that variety, where \mathbb{G}_a denotes the additive group over the ground field. Given a variety, its Makar-Limanov invariant is the ring of all absolute constants. We will see a geometric interpretation of Makar-Limanov's result that the coordinate function x on the cubic threefold given by $x + x^2y + z^3 + t^2 = 0$ is an absolute constant, and I hope to give applications of the underlying geometric construction of this interpretation also to other situations in the study of \mathbb{G}_a -actions on affine varieties.

- Masayoshi MIYANISHI :

Title: **Remarks on the generalized Jacobian conjecture for \mathbb{A}^2/G**

Abstract: Let G be a small finite subgroup of $\mathrm{GL}(2, \mathbb{C})$ and let $X = \mathbb{A}^2/G$. Let $\varphi : X \rightarrow X$ be a quasi-étale endomorphism which is, by definition, étale on the smooth part X° of X . We consider the generalized Jacobian conjecture for X which asserts that φ is an automorphism. The quotient morphism $\pi : \mathbb{A}^2 \rightarrow X$ is a quasi-universal covering. Then φ lifts to a (G, χ) -equivariant étale endomorphism $\tilde{\varphi} : \mathbb{A}^2 \rightarrow \mathbb{A}^2$, i.e., $\tilde{\varphi}(O) = O$ and $\tilde{\varphi}(gx) = \chi(g)\tilde{\varphi}(x)$ for $x \in \mathbb{A}^2$ and $g \in G$, where $\chi : G \rightarrow G$ is a group automorphism. If $\chi = \mathrm{id}$, then we say that $\tilde{\varphi}$ is G -equivariant. Let \hat{X} be the minimal resolution of singularity of X . Then φ extends

to an étale endomorphism $\widehat{\varphi} : \widehat{X} \rightarrow \widehat{X}$ such that $\widehat{\varphi}$ restricted onto the exceptional loci is an automorphism provided $\widetilde{\varphi}$ is G -equivariant. On the other hand, the smooth part X° has the standard \mathbb{A}_*^1 -fibration $p : X^\circ \rightarrow \mathbb{P}^1$. If φ preserves the standard \mathbb{A}_*^1 -fibration p then both φ and $\widetilde{\varphi}$ are automorphisms. We look for sufficient conditions with which φ preserves the standard \mathbb{A}_*^1 -fibration or its substitute.

- Adrien DUBOULOZ:

Title: **Exotic \mathbb{A}^1 -contractible affine threefolds**

Abstract: The so-called Koras-Russell threefolds are a family of topologically contractible rational smooth complex affine threefolds which played an important role in the linearization problem for multiplicative group actions on the affine 3-space. They are known to be all diffeomorphic to the 6-dimensional euclidean space, but it was shown by Makar-Limanov in the nineties that all of them are not algebraically isomorphic to the affine 3-space. It is however not known whether they are stably isomorphic or not to an affine space. Recently, Hoyois, Krishna and Østvær proved that many of them become contractible in the \mathbb{A}^1 -homotopy category of Morel and Voevodsky after some finite suspension with the pointed projective line. In this talk, I will explain how additional geometric properties related to additive group actions on such varieties allow to conclude that a large class of them are actually \mathbb{A}^1 -contractible (Joint work with Jean Fasel, Université Grenoble-Alpes).

★ 4th March (Saturday)

- Masaru NAGAOKA:

Title: **Contractible affine threefolds in smooth Fano threefolds**

Abstract: By the contribution of M. Furushima, N. Nakayama, Th. Peternell and M. Schneider, it is completed to classify all compactifications of the affine 3-space \mathbb{A}^3 with Picard number one. As a similar question, T. Kishimoto raised the problem to classify all triplets $(V, U, D_1 \cup D_2)$ which consist of smooth Fano threefolds V of Picard number two, contractible affine threefolds U as open subsets of V , and the complements $D_1 \cup D_2 = V \setminus U$. We determine all Fano threefolds V which appear in such triplets when the log canonical divisor $K_V + D_1 + D_2$ is linearly equivalent to zero.

- Karol PALKA:

Title: **How to classify \mathbb{Q} -homology planes of general type ?**

Abstract: A complex algebraic surface with vanishing reduced Betti numbers is called a " \mathbb{Q} -homology plane". These surfaces and their relations with various important questions in affine geometry have been the subject of intense studies since the 80's. The ones not of log general type have been classified using structure theorems from the theory of open surfaces. We show a new approach to the classification of \mathbb{Q} -homology planes of log general type using the theory of almost minimal models with fractional coefficients and a reduction to primitive \mathbb{Q} -homology planes.

- Joonyeong WON:

Title: K -stability of smooth del Pezzo surfaces

Abstract: We introduce new invariant, delta-invariant for K -stability of Fano variety due to Y. Odaka and K. Fujita. By using the invariant, in algebro-geometric way, we completely determine whether smooth del Pezzo surfaces are K -stable.

- Takeru FUKUOKA:

Title: On the existence of almost Fano threefolds with del Pezzo fibrations

Abstract: We say that a projective manifold is almost Fano if its anti-canonical divisor is nef and big but not ample. In this talk, we consider almost Fano 3-folds of Picard rank 2 with del Pezzo fibration. By Jahnke- Peternell-Radloff and Takeuchi, the numerical conditions of such 3-folds were classified. Among them, there exists 10 cases such that the existence of examples of these was not proved. The main result of this talk is to show the existence of examples belonging to each of 10 classes. Except for one case, the degree of del Pezzo fibrations are 6. We will observe one of the reason of difficulty constructing del Pezzo fibrations of degree 6 and that every almost Fano del Pezzo fibration of degree 6 with specific anti-canonical volume can be embedded into some higher dimensional del Pezzo fibration as a relative linear section.

- Takuzo OKADA:

Title: On stable rationality of del Pezzo fibrations

Abstract: I will talk about stable nonrationality of (many) del Pezzo fibrations over projective space. Stable rationality is an intermediate notion between rationality and unirationality. After explaining briefly about these notion, I will explain the main idea of the proof which is the combination of specialization (mod p reduction) argument due to Voisin, Colliot-Thélène and Pirutka, and Kollár's technique on inseparable cyclic covering spaces. This is a joint work with Igor Krylov.

★ 5th March (Sunday)

- Hiroyuki ITO:

Title: On quasi-hyperelliptic fibrations

Abstract: There are special surfaces in positive characteristic, called quasi-elliptic surfaces, which have quasi-elliptic fibration. The quasi-elliptic fibration is a fibration structure that almost all fibers are irreducible singular curves of arithmetic genus 1. Especially, unirational quasi-elliptic surfaces are related with a form of the affine line. Many arithmetic and geometric properties are studied. In the talk, we consider the higher genus analogy of this surfaces, that is, surfaces with quasi-hyperelliptic fibration structure. The quasi-hyperelliptic fibration is a fibration structure that almost all fibers are irreducible singular curves of arithmetic genus > 1 . These fibrations are also related with a form of the affine line. And as for the case of quasi-elliptic fibration, we can study arithmetic properties such as Mordell-Weil groups, and geometric properties such as classification of degenerate fibers.

- Pierre-Marie POLONI:

Title: The Jonquière's subgroup is a Borel subgroup

Abstract: The Jonquière's subgroup is the group B_n of triangular polynomial automorphisms of the complex affine n -space. By analogy with linear groups, many authors refer to it as the triangular "Borel" subgroup of $\text{Aut}(\mathbb{A}^n)$. In this talk, we will show that the quotation marks could be dropped. Indeed, we will prove that B_n is maximal among all (connected) solvable subgroups of $\text{Aut}(\mathbb{A}^n)$, thus a Borel subgroup of $\text{Aut}(\mathbb{A}^n)$, when the latter is viewed as an ind-group. We will also consider the following question: Are Borel subgroups of $\text{Aut}(\mathbb{A}^n)$ all conjugate? This is joint work with Jean-Philippe Furter.

- Shigeru KURODA:

Title: Hilbert's Fourteenth Problem and field modifications

Abstract: Let k be a field, and $k(\mathbf{x})$ the field of fractions of the polynomial ring $k[\mathbf{x}] = k[x_1, \dots, x_n]$. For an intermediate field $k \subset L \subset k(\mathbf{x})$, Hilbert's Fourteenth Problem asks whether the k -algebra $A := L \cap k[\mathbf{x}]$ is finitely generated. If the transcendence degree of A is at most two over k , then A is always finitely generated by Zariski. On the other hand, the first counterexample was given by Nagata in 1958. In this talk, we discuss the following type of problem: Let $k \subset M \subsetneq k(\mathbf{x})$ be an intermediate field with $M \cap k[\mathbf{x}]$ of transcendence degree at least three over k . Does there exist $\sigma \in \text{Aut}_k k(\mathbf{x})$ such that $\sigma(M)$ is a counterexample to Hilbert's Fourteenth Problem? We give an affirmative answer to a stable version of this problem.

- Akinari HOSHI:

Title: Rationality problem for fields of invariants

Abstract: The aim of this talk is to give a survey of recent developments in the rationality problem for fields of invariants, e.g. Noether's problem, rationality problem for algebraic tori, rationality problem of quasi-monomial actions. I will give some explicit examples of (non-)stably (resp. retract) rational fields which are weaker concepts than the rationality. Negative results may be obtained by using (stable) birational invariants, e.g. flabby class, unramified Brauer (cohomology) group which is an avatar of the Artin-Mumford invariant $H^3(X, \mathbb{Z})_{\text{tors}}$.