# THE 14TH AFFINE ALGEBRAIC GEOMETRY MEETING

#### ABSTRACTS OF TALKS

 $\star$  5th March (Saturday)

• Ryuji Tanimoto:

# Title: On triangular automorphisms of the polynomial ring in three variables over a field of positive characteristic

Abstract: Let k be a field of positive characteristic p and let k[x, y, z] be the polynomial ring in three variables over k. It is an interesting problem to construct automorphisms of k[x, y, z]of order p. In this talk, we give an expression for the triangular automorphisms of k[x, y, z]of order p up to conjugation by automorphisms of k[x, y, z].

• Takanori Nagamine:

#### Title: Derivations and closed polynomials over domains

Abstract: For a non-constant polynomial f in the polynomial ring  $R^{[n]}$  in n variables over an integral domain R satisfying  $Q(R)[f] \cap R^{[n]} = R[f]$ , we give necessary and sufficient conditions for f to be a closed polynomial.

• Isac Hedén:

#### Title: Extensions of principal additive bundles over a punctured surface

Abstract: We study complex affine  $\mathbb{G}_a$ -threefolds X such that the restriction of the quotient morphism  $\pi: X \to S$  to  $\pi^{-1}(S_*)$  is a principal  $\mathbb{G}_a$ -bundle, where  $S_*$  denotes the complement of a closed point in S. Changing the point of view, we look for affine extensions of  $\mathbb{G}_a$ principal bundles over punctured surfaces, i.e affine varieties that are obtained by adding an extra fiber to the bundle projection over the puncture. The variety  $SL_2$  will be of special interest and a source of many examples.

• Hiroyuki Ito:

#### Title: Pseudo-derivations and wild group scheme quotient singularities

Abstract: When one treats 2-dimestional quotient singularities in positive characteristic, most difficult and mysterious ones are wild quotient singularities which are the quotient by finite groups of order divisible by the characteristic. On the other hand, there are some other kinds of wild quotient singularities which are the quotient by finite flat group schemes of length divisible by the characteristic. In this talk, we consider the quotients by pseudo-derivations which are derivation-like endomorphism, and give some family of group scheme quotients as deformation families which connect these two kinds of singularities, that is, Frobenius sandwich singularities and Artin-Schreier sandwich singularities, under some conditions. As an application, this explains the mathematical reason for non-tautness of many singularities of purely inseparable type. Finally, we consider the problem whether rational double points in positive characteristic are group scheme quotient singularities or not.

 $\star$  6th March (Sunday)

• Shigeru Kuroda:

# Title: Stably co-tame polynomial automorphisms over commutative rings

Abstract: Derksen's theorem says that the tame subgroup of the automorphism group of the polynomial ring in three or more variables is generated only by affine automorphisms and a certain elementary automorphism. There are several generalizations of this result. Edo defined an automorphism F to be co-tame if the subgroup generated by affine automorphisms and F contains the tame subgroup, and proved that Nagata's automorphism is co-tame. In this talk, we say that an automorphism F in n variables is stably co-tame if the subgroup generated by affine automorphisms in n + 1 variables and F contains the tame subgroup in n variables, where we regard automorphisms in n variables as those in n + 1 variables in a natural way. Our main result gives a necessary and sufficient condition for stably co-tameness in the case where the coefficient ring of the polynomial ring contains an infinite field.

• Takeshi Takahashi:

# Title: Quasi-Galois points for a plane algebraic curve

Abstract: A point P in projective plane is said to be "quasi-Galois" for a plane curve C if the curve C has a non-trivial birational transformation  $\sigma$  which satisfies  $\pi_P \circ \sigma = \pi_P$ , where  $\pi_P$  is the projection from P. I will explain some theorems stated in our paper "Quasi-Galois points" by S. Fukasawa, K. Miura and T. Takahashi (arXiv:1505.00148).

• Fumio Sakai:

# Title: Weierstraß weight, Theorems of Chowla and White, Terminal Lemma

Abstract: We classified d-cyclic covers of the projective line which has a ramification point with minimum Weierstraß weight. In the proof, we employed a combinatorial number theory such as the theorem of Chowla or the theorem of White in late 1960's, which was also used in Terminal Lemma in 1980's. This is a joint work with Nam Wangyu and Masumi Kawasaki.

• Kiwamu Watanabe:

#### Title: Fano manifolds with nef tangent bundle

Abstract: As a generalization of Mori's characterization of the projective space, Campana and Peternell conjectured that the only Fano manifolds with nef tangent bundle are rational homogeneous. In this talk, we review recent developments on the conjecture, with special attention to the results obtained in my joint work with R. Muñoz, G. Occhetta, L E. Solá Conde and J. Wiśniewski.

• Stéphane Druel:

#### Title: On regular foliations with nef anti-canonical bundle

Abstract: In recent years, techniques from the minimal model program have been successfully applied to the study of global properties of holomorphic foliations. In this talk I will explain that the anti-canonical bundle of a regular holomorphic foliation on a complex projective manifold cannot be nef and big. Then I will address codimension one regular foliations whose anti-canonical bundle is nef with maximal Kodaira dimension.

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# $\star$ 7th March (Monday)

• Kento Fujita:

# Title: Optimal bounds for the volumes of Kähler-Einstein Fano manifolds

Abstract: We show that any *n*-dimensional Kähler-Einstein Fano manifold X satisfies that the anti-canonical volume is less than or equal to the value  $(n + 1)^n$ . Moreover, the equality holds if and only if X is isomorphic to the projective space.

• Takuzo Okada:

# Title: On log canonical thresholds of birationally rigid Fano threefolds

Abstract: The global log canonical threshold of a Fano variety is an algebraic counterpart of Tian's alpha invariant. I will explain a recent work on the computations of global log canonical thresholds of certain birationally rigid Fano threefolds. As a main application, the existence of Kähler-Einstein metric on those Fano varieties follows. This is a joint work with In-Kyun Kim and Joonyeong Won.

• Hiromichi Takagi:

# Title: On classification of prime $\mathbb{Q}$ -Fano threefolds with only 1/2(1, 1, 1)-singularities of genus $\leq 1$

Abstract: After a long time, I revisit the classification problem of prime  $\mathbb{Q}$ -Fano threefolds with only 1/2(1,1,1)-singularities. Previously I almost classified those with genus greater than or equal to 2. Now I am working on the classification of those with genus less than 2. In my talk, I will explain a general method to restrict several invariants of those. I also describe in detail a few examples of those which entail classical projective geometry.

#### • Masayoshi Miyanishi:

# Title: Singular fibers of $\mathbb{A}^1$ -fibrations on affine threefolds

Abstract: Let X be a smooth factorial affine 3-fold with a  $\mathbb{G}_a$ -action and let  $q: X \to Y$  be the quotient morphism. Then every fiber of q has dimension one and Y has at worst quotient singularities. By comparing  $H_c^i(X;\mathbb{Z})$  with  $H_c^i(X^{\mathbb{G}_a};\mathbb{Z})$  à la Bialynicki-Birula, where  $H_c^i$  is the integral cohomology with compact support and  $X^{\mathbb{G}_a}$  is the  $\mathbb{G}_a$ -fixed point locus of X, we have shown that every singular fiber F of q is a disjoint union of contractible curves and asked if it is indeed a disjoint union of the  $\mathbb{A}^1$ . We answer this question affirmatively in three different situations, each of which has its own interest.

- (1) If Y is isomorphic to the affine plane  $\mathbb{A}^2$  and  $F_0$  is an irreducible component of multiplicity one, then  $F_0$  is isomorphic to  $\mathbb{A}^1$ .
- (2) Assign weights a, b, c to variables x, y, z respectively of a polynomial ring  $\mathbb{k}[x, y, z]$ , where a, b, c are pairwise coprime integers > 1. Let  $\mathbb{G}_a$  act on  $X = \operatorname{Spec} \mathbb{k}[x, y, z]$ via a homogeneous locally nilpotent derivation  $\delta$  with respect to this grading. Then every fiber component of F is isomorphic to  $\mathbb{A}^1$ .
- (3) Finally we have the following general result which includes the above two cases and depends on a Kollár's theorem.

**Theorem 1.** Let X be a smooth affine 3-fold with a  $\mathbb{G}_a$ -action and let  $q: X \to Y$  be the quotient morphism. If  $F_0$  is a fiber component of dimension one of q, then  $F_0$  is isomorphic to  $\mathbb{A}^1$ .

This is a joint work with R.V. Gurjar, M. Koras, K. Masuda and P. Russell.

• Ivan Cheltsov:

# Title: Cylinders in del Pezzo surfaces

Abstract: For a projective variety X and an ample divisor H on it, an H-polar cylinder in X is an open ruled affine subset whose complement is a support of an effective  $\mathbb{Q}$ -divisor  $\mathbb{Q}$ -rationally equivalent to H. I will show how to prove existence and non-existence of H-polar cylinders in smooth and mildly singular del Pezzo surfaces (for different polarizations). This is a joint work with Jihun Park and Joonyeong Won.

- $\star$  8th March (Tuesday)
- Tomoaki Ohta:

# Title: Some remarks on tameness and wildness of automorphisms of $\mathbb{A}^3$

Abstract: Recently, several researchers have investigated multidegrees of tame and wild automorphisms of  $\mathbb{A}^3$  by applying the Shestakov-Umirbaev theory. We will give some remarks and examples related to their results.

• Adrien Dubouloz:

# Title: Étale endomorphisms of affine surfaces

Abstract: A smooth complex variety satisfies the Generalized Jacobian Conjecture if every of its étale endomorphism is proper. For varieties of nonzero Euler characteristic properness is actually equivalent to the invertibility of the corresponding étale endomorphism. The conjecture is of course especially interesting for affine varieties of negative logarithmic Kodaira dimension and with trivial rational homology, because of their similarity to affine spaces. In this talk, I will review existing general positive results for rational homology planes and then present a series of examples of affine pseudo-planes of negative Kodaira dimension for which the Generalized Jacobian Conjecture fails very badly. (Joint work in progress with K. Palka, (IMPAN Warsaw)).

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