THE 18TH AFFINE ALGEBRAIC GEOMETRY MEETING

ABSTRACTS of TALKS

⋆ 5th March (Thursday)

• Takanori NAGAMINE (Niigata University):

Title: **A family of strongly invariant algebras**

Abstract: Let k be a field and let R be a k-algebra. We say that R is strongly invariant if for any integer $n \geq 0$ and k-algebra S, $R[x_1, \ldots, x_n] \cong S[y_1, \ldots, y_n]$ implies $R \cong S$. In this talk, we consider the following k-algebra: $R_{(m,n)} = k[x, y_1, \ldots, y_n, u]/(ut - (x - 1))$, where $t = x^m y_1 \cdots y_n - 1$ and $m, n \geq 2$. We show that $R_{(m,n)}$ is strongly invariant by using terms of non-negative degree functions, and $R_{(m,n)} \cong R_{(r,s)}$ if and only if $(m, n) = (r, s)$. Moreover, $V_{(m,n)} = \mathrm{Spec}(R_{(m,n)})$ is smooth factorial $(n + 1)$-dimensional variety with trivial units of $\bar{\kappa}(V_{(m,n)}) \leq 0$, where $\bar{\kappa}$ means the logarithmic Kodaira dimension. In particular, if $n = 1$, then $\bar{\kappa}(V_{(m,1)}) = 0$.

• Riku KUDO (Waseda University):

Title: **Generalized Zariski cancellation problem and principal $\mathbb{G}_a$-bundles**

Abstract: Generalized Zariski cancellation problem asks whether or not $V \times \mathbb{A}^1 \simeq W \times \mathbb{A}^1$ implies $V \simeq W$ for varieties V and W. Counter examples for this problem have been constructed as principal $\mathbb{G}_a$-bundles over prevarieties. In 2007, Dryło showed that vector bundles over non $\mathbb{A}^1$-uniruled affine varieties have the cancellation property. In this talk I will explain a slight generalization of Dryło's lemma used to show the above theorem and show the following theorem; if an affine variety V has a principal $\mathbb{G}_a$-bundle structure over a non $\mathbb{A}^1$-uniruled prevariety X, then for an affine variety W, $V \times \mathbb{A}^1 \simeq W \times \mathbb{A}^1$ if and only if W has a principal $\mathbb{G}_a$-bundle structure over X.

• Ryuji TANIMOTO (Shizuoka University):

Title: **Exponential matrices of size five-by-five**

Abstract: In this talk, we give an overlapping classification of exponential matrices of size five-by-five in positive characteristic p. Using this classification, we can give an overlapping classification of five-dimensional modular representations of elementary abelian p-groups.

• Buddhadev HAJRA (Indian Institute of Technology, Bombay):

Title: **Zariski's finiteness theorem and properties of some rings of invariants**

Abstract: In this talk I will present a short proof of a special case of O. Zariski's result about finite generation in connection with Hilbert's 14th problem using a new idea. This result is useful for invariant subrings of unipotent or connected semisimple groups. The next result I will talk about is a stronger form of one well-known result by A. Tyc. This result proves that the quotient space under a regular $\mathbb{G}_a$-action on an affine space over the field of complex numbers has at most rational singularities, under an assumption about the quotient morphism. I will also sketch the main idea of the proof of a result which is an analogue of M. Miyanishi's result for the ring of invariants of a $\mathbb{G}_a$-action on $R[X, Y, Z]$ for an affine Dedekind domain R. This proof involves some classical topological methods. This is a joint work with R.V. Gurjar and Sudarshan R. Gurjar.

• Shigeru KURODA (Tokyo Metropolitan University):

Title: **Finitely generated polynomial subalgebras without finite SAGBI bases**

Abstract: SAGBI (Subalgebra Analogue to Groebner Bases for Ideals) bases are defined for subalgebras of a polynomial ring, similarly to Groebner bases for ideals. Groebner bases are always finite sets due to Hilbert's basis theorem, but finitely generated subalgebras of a polynomial ring do not always have finite SAGBI bases. In this talk, we give a new class of finitely generated subalgebras without finite SAGBI bases. We construct such subalgebras using strongly convex rational polyhedral cones.

⋆ 6th March (Friday)

• Masaru NAGAOKA (University of Tokyo):

Title: **$\mathbb{G}_a^3$-structures on del Pezzo fibrations**

Abstract: A $\mathbb{G}_a^n$-structure on a variety X is a $\mathbb{G}_a^n$-action on X with the dense open orbit isomorphic to $\mathbb{G}_a^n$. Projective varieties with $\mathbb{G}_a^n$-structures are considered as equivariant compactifications of the affine n-space. Hassett-Tschinkel initiated the study of $\mathbb{G}_a^n$-structures, and they and Huang-Montero completed the determination of smooth Fano 3-folds admitting $\mathbb{G}_a^3$-structures. In this talk, we discuss the determination of del Pezzo fibrations admitting $\mathbb{G}_a^3$-structures.

• Pedro MONTERO (Universidad Tecnica Santa Maria, UTFSM, Valparaiso):

Title: **Equivariant compactifications of the vector group into smooth Fano manifolds**

Abstract: Thanks to the recent works of Birkar, we know that there are only finitely many families of midly singular Fano varieties in every fixed dimension. However, even for smooth Fano varieties, there is no complete classification in dimension greater than or equal to four. Because of that, it is natural to impose some geometric conditions in order to try to classify those varieties. In this talk we will study the geometry of Fano manifolds that are obtained as equivariant compactifications of the vector group. Historically, Hassett and Tschinkel iniciated the study of the geometry of those varieties, which enjoy some nice arithmetic properties such as the Batyrev-Manin principle (concerning the asymptotical distribution of rational points). After giving some general properties and examples of such varieties, we will discuss how the works of Hassett and Tschinkel, Kishimoto, Arzhantsev et al. fit together with the classification of Fujita, Mori and Mukai in order to allow us to give a complete classification of " additive Fano manifolds " when the dimension is 3 (joint work with Zhizhong Huang) and when the Fano index is high (joint work with Baohua Fu).

• Masayoshi MIYANISHI (Kwansei Gakuin University):

Title: **Generalized Jacobian Conjecture for $\mathbb{A}^2/G$ and $\mathbb{P}^2 \setminus C$**

Abstract: Let G be a small finite subgroup of $\mathrm{GL}(2,\mathbb{C})$ and let $\widetilde{\varphi} : \mathbb{A}^2 \to \mathbb{A}^2$ be a G-equivariant étale endomorphism of the affine plane. Then $\widetilde{\varphi}$ induces a quasi-étale endomorphism φ on the singular quotient surface $\mathbb{A}^2/G$ whose smooth part X° has the standard $\mathbb{A}^1_*$-fibration $p : X^\circ \to \mathbb{P}^1$. If φ preserves the standard $\mathbb{A}^1_*$-fibration p then both φ and $\widetilde{\varphi}$ are automorphisms. We look for the condition with which φ preserves the standard $\mathbb{A}^1_*$-fibration. So far we have proved the GJC in the case where G is cyclic and its action is given ${}^\zeta(x,y) = (\zeta x, \zeta y)$. We also consider the conjecture for the complement $\mathbb{P}^2 \setminus C$ for a cubic curve C which is not necessarily smooth or irreducible.

• Karol PALKA (Polish Academy of Sciences, Warsaw):

Title: **Generalized Jacobian Conjecture and deformations**

Abstract:

• Tomasz PEŁKA (University of Bern, Bern):

Title: **$\mathbb{Q}$-homology planes satisfying the Negativity Conjecture**

Abstract: A smooth complex normal algebraic surface S is a $\mathbb{Q}$-homology plane if $H_i(S,\mathbb{Q}) = 0$ for $i > 0$. This holds for example if S is a complement of a rational cuspidal curve in $\mathbb{P}^2$. The Negativity Conjecture of K. Palka asserts that for a smooth completion (X,D) of S, $\kappa(K_X + \frac{1}{2}D) = -\infty$. Assume that S is of log general type, otherwise the geometry is well understood. It turns out that all S satisfying the Negativity Conjecture can be arranged in finitely many discrete families, each obtainable in a uniform way,, as expected by tom Dieck and Petrie, from certain arrangements of lines and conics on $\mathbb{P}^2$. As a consequence, all such S satisfy the Strong Rigidity Conjecture of Flenner and Zaidenberg; and all their automorphism groups are subgroups of S_3. To illustrate this rigidity, I will show how to construct all rational cuspidal curves (with complements of log general type, satisfying the Negativity Conjecture) inductively, by iterating quadratic Cremona maps.

⋆ 7th March (Saturday)

• Katsuhiko OKUMURA (Waseda University):

Title: **SNC log symplectic structures on Fano products**

Abstract: In this talk, we classify SNC log symplectic structures on the product of Fano varieties with cyclic Picard group. A log symplectic structure is a Poisson structure with the reduced degeneracy divisor, and SNC means that the degeneracy divisor has only simple normal crossing singularity. In 2014, Lima and Pereira classified such structures on the Fano varieties of Picard rank 1. And then Pym gave another proof. I will introduce Pym's method and construction of Poisson structures on the projective spaces from that on the affine spaces.

• Kenta HASHIZUME (University of Tokyo):

Title: **On minimal model theory for log canonical pairs with big boundary divisors**

Abstract: In the birational geometry, minimal model theory predicts that all normal projective varieties have nice canonical divisors after birational modifications. Currently, we often consider minimal model theory for Kawamata log terminal pairs or log canonical pairs, which are pairs of a normal variety and a divisor on it with mild singularities. By Birkar, Cascini, Hacon and McKernan, minimal model theory for Kawamata log terminal pairs was established in some important cases. In this talk, I introduce their results and explain generalizations of their results to log canonical pairs.

• Masatomo SAWAHARA (Saitama University):

Title: **Cylinders in canonical del Pezzo fibrations**

Abstract: It is known that by the work due to Dubouloz and Kishimoto a del Pezzo fibration $\pi : V \to W$ of degree d contains a vertical cylinder if and only if $d \geqq 5$ and the generic fiber V_η, which is a smooth del Pezzo surface of Picard rank one defined over the field $\mathbb{C}(\eta) = \mathbb{C}(W)$ of functions of the base variety W, admits a $\mathbb{C}(W)$-rational point. Instead, in this talk, we will observe a del Pezzo fibration $\pi : V \to W$ of degree d with *canonical* singularities and look for a criterion for V to contain vertical cylinders with respect to π. The problem is reduced to the existence of cylinder found on the generic fiber $V_\eta = \pi^{-1}(\eta)$, which is a normal Gorenstein del Pezzo surface of Picard rank one defined over the field $\mathbb{C}(\eta) = \mathbb{C}(W)$. We shall give a complete answer about the existence of vertical cylinder found in V with respect to π depending on the degree d and type of singularities. The case of $d \leqq 2$ is especially complicated, so that we will devote ourselves mainly to the case of $d \leqq 2$ in the talk.