THE 16TH AFFINE ALGEBRAIC GEOMETRY MEETING

ABSTRACTS OF TALKS

⋆ 8th March (Thursday)

• Takeshi TAKAHASHI (Niigata University):

Title: **Number of weak Galois Weierstraß points with semigroup $< a, b >$**

Abstract: Let C be a nonsingular projective curve of genus > 1 over an algebraically closed field of characteristic 0. For a point P in C, the Weierstraß semigroup $H(P)$ is defined as the set of non-negative integers n for which there exists a rational function f on C such that the order of the pole of f at P is equal to n, and f is regular away from P. A point P in C is referred to as a weak Galois Weierstraß point if P is a Weierstraß point and there exists a Galois morphism from C to the projective line such that P is a total ramification point of the morphism. Prof. Komeda and I are investigating weak Galois Weierstraß points whose Weierstraß semigroups are generated by two positive integers, as a generalization of studies on Galois points for plane curves defined by Prof. Yoshihara. In my talk, I will introduce some of these results.

• Yuta KAMBE (Saitama University):

Title: **On the moduli space of reduced Gröbner bases**

Abstract: For a given monomial ideal J and a given monomial order $<$ on a polynomial ring S, the moduli space of reduced Gröbner bases in S whose initial ideal is J is determined. I will talk about a construction and an application of such moduli spaces.

• Yoshinori GONGYO (The University of Tokyo):

Title: **Nef anti-canonical divisors and rationally connected fibrations**

Abstract: We study the Iitaka–Kodaira dimension of nef relative anti-canonical divisors by using semi-positivity theorems. Thus we give affirmative answers for some two questions. This is a joint work with Sho Ejiri.

• Kei MIURA (National Institute of Technology, Ube College):

Title: **Birational transformations belonging to Galois points**

Abstract: In this talk, we study birational transformations belonging to Galois points. For a certain plane quartic curve, we determine the number of Galois points. Then, we obtain birational transformations belonging to these points. In fact, we see that they can be extended to Cremona transformations. In particular, we determine their conjugacy class and show that they are all conjugate to linear transformations.

⋆ 9th March (Friday)

• Kiwamu WATANABE (Saitama University):

Title: **Varieties with nef diagonal**

Abstract: For a smooth projective variety X, we consider when the diagonal Δ_X is nef as a cycle on $X \times X$. In particular, we give a classification of complete intersections and smooth del Pezzo varieties where the diagonal is nef. We also study the nefness of the diagonal for spherical varieties. This is a joint work with Taku Suzuki.

• Masayoshi MIYANISHI (Kwansei Gakuin University):

Title: **Geometry of Artin-Schreier coverings**

Abstract: In positive characteristic, a finite morphism $q : Y \to X$ of algebraic varieties is called an Artin-Schreier covering if the extension of function fields $k(X) \subset k(Y)$ is an Artin-Schreier covering, i.e., a Galois covering of group $\mathbb{Z}/p\mathbb{Z}$. We consider the case where X and Y are smooth projective surfaces. There is a pioneering work by Y. Takeda [J. Math. Soc. Japan, vol 41, No.3, 1989], where basic results and Enriques classification of surfaces obtained as Artin-Schreier coverings of simple type are outlined. But it seems that the details have to be exploited and can lead to an interesting geometry. This is a report of a review that the speaker has made recently.

• Adrien DUBOULOZ (Université de Bourgogne):

Title: **Algebraic models of the real affine plane**

Abstract: An algebraic model of the plane $\mathbb{R}^2$ is a smooth complex surface S defined over the real numbers, with trivial rational singular homology and real locus homeomorphic to $\mathbb{R}^2$. Two such models S and S' are considered to be "the same" if they are "birationally diffeomorphic", that is, there exists a birational map $S \dashrightarrow S'$ defined over the real numbers which induces a diffeomorphism between the real loci of S and S'. In this talk, I will explain how to construct a variant of the logarithmic Kodaira dimension which is an invariant of such surfaces up to birational diffeomorphisms. An application is the existence of infinitely many pairwise non birationally diffeomorphic algebraic models of $\mathbb{R}^2$, a situation which contrasts with the analogue problem for rational algebraic models of compact differentiable real surfaces. If time permits, I will also indicate some relation between the classification of such models and a birational variant of the classical Abhyankar-Moh embedding problem.

• Masayuki KAWAKITA (RIMS):

Title: **Minimal log discrepancies on smooth threefolds**

Abstract: The minimal log discrepancy is an important invariant of singularities in the minimal model program. We will discuss several equivalent conjectures on the minimal log discrepancies on smooth threefolds.

⋆ 10th March (Saturday)

• Pierre-Marie POLONI (The University of Bern):

Title: **Affine plane bundles over the punctured affine plane**

Abstract: An $\mathbb{A}^2$-fibration is a flat morphism between complex algebraic varieties whose fibers are isomorphic to the complex affine plane $\mathbb{A}^2$. In this talk, we study explicit families $f : \mathbb{A}^4 \to \mathbb{A}^2$ of $\mathbb{A}^2$-fibrations over $\mathbb{A}^2$. The famous Dolgachev-Weisfeiler conjecture predicts that such fibrations are in fact all isomorphic to the trivial bundle over $\mathbb{A}^2$. Our aim is to develop tools for verifying that this conjecture holds true in some particular examples. For instance, we will see that the $\mathbb{A}^2$-fibration induced by the second Vénéreau polynomial is trivial. Our strategy is inspired by a previous work of Kaliman and Zaidenberg and consists in first showing that the considered fibrations $f : \mathbb{A}^4 \to \mathbb{A}^2$ have a fiber bundle structure when restricted over the punctured plane $\mathbb{A}^2 \setminus \{(0, 0)\}$. This is joint work in progress with Jérémy Blanc.

• Ivan CHELTSOV (The University of Edinburgh):

Title: **Kähler-Einstein Fano threefolds of degree 22**

Abstract: This talk is about smooth Fano threefolds of Picard rank 1 and degree 22 that admit an action of a multiplicative group. These threefolds form a one-parameter family that contains the famous Mukai-Umemura threefold as a special member. In 2008, Donaldson proved that Mukai-Umemura threefold is Kähler-Einstein. He wrote: "The Mukai-Umemura manifold has $\tau = 1$. When τ is close to 1 we have seen that the corresponding manifold admits a Kähler-Einstein metric. It seems likely that this true for all τ but, as far the author is aware, this is not known. It seems an interesting test case for future developments in the existence theory". Here τ is a parameter in the moduli of these threefolds. I will show how to prove that all of them are Kähler-Eisntein except for possibly two threefolds in this family. If time permits, we discuss what to do with the remaining two threefolds. This is a joint work with Costya Shramov (Moscow).

• Osamu FUJINO (Osaka University):

Title: **On semi-log canonical Fano pairs**

Abstract: It is well known that every Fano manifold is simply connected and is rationally chain connected. This result was already generalized for Fano log pairs with log canonical singularities. The notion of semi-log canonical singularities was introduced by Kollár–Shepherd-Barron in order to investigate deformations of surface singularities and compactifications of moduli spaces for surfaces of general type. We have already known that the appropriate singularities to permit on the varieties at the boundaries of moduli spaces of higher-dimensional varieties are semi-log canonical. In this talk, I would like to explain the following theorem:

Theorem 0.1. *Let (X, Δ) be a connected projective semi-log canonical pair such that $-(K_X + \Delta)$ is ample. Then X is simply connected and is rationally chain connected.*

We note that X may be non-normal and reducible. This talk is based on the joint work with Wenfei Liu.

• Kento FUJITA (RIMS):

Title: **K-stability of log Fano hyperplane arrangements**

Abstract: We completely determine which log Fano hyperplane arrangements are uniformly K-stable, K-stable, K-polystable, K-semistable or not.

• Takuzo OKADA (Saga University):

Title: **On birational rigidity of singular del Pezzo fibrations of degree 1**

Abstract: A del Pezzo fibration (or more generally, a Mori fiber space) is said to be birationally rigid if it cannot be birationally transformed into a Mori fiber space which is not isomorphic to the original del Pezzo fibration. Birational rigidity of nonsingular del Pezzo fibrations of degree 1 is completely understood by Pukhlikov and Grinenko. However almost nothing is known for singular case. In this talk I will report some results of on-going project studying birational rigidity of del Pezzo fibrations of degree 1 with terminal quotient singularities.

⋆ 11th March (Sunday)

• Isac HEDÉN (The University of Warwick):

Title: **Extensions of principal additive bundles over a punctured surface**

Abstract: We study complex affine $\mathbb{G}_a$-threefolds X such that the restriction of the quotient morphism $\pi : X \to S$ to $\pi^{-1}(S_*)$ is a principal $\mathbb{G}_a$-bundle, where $S_* = S \backslash \{o\}$ denotes the complement of a closed point o in S and $\mathbb{G}_a$ denotes the additive group over the field of complex numbers. Changing the point of view, we look for affine extensions of $\mathbb{G}_a$-principal bundles over punctured surfaces, i.e., affine varieties that are obtained by adding an extra fiber to the bundle projection over o. Special attention will be given to the case where X is smooth, the $\mathbb{G}_a$-action on X is proper and $\pi^{-1}(o) = \mathbb{A}^2$ is the affine plane.

• Takeru FUKUOKA (The University of Tokyo):

Title: **Relative linear extensions of sextic del Pezzo fibrations**

Abstract: An extremal contraction from a non-singular projective 3-fold onto a smooth curve is called a del Pezzo fibration. It is classically known that every del Pezzo surface S is a (weighted) complete intersection of a certain Fano variety. In order to study del Pezzo fibrations, it is important to relativize such descriptions for those. The main result of this talk shows that the sextic del Pezzo fibrations are relative linear sections of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$-fibrations and $\mathbb{P}^2 \times \mathbb{P}^2$-fibrations, which are constructed as Mori fiber spaces with smooth total space. As an application, we will classify the singular fibers of sextic del Pezzo fibrations.