Gil Kalai

Hebrew University

Advances and challenges in the combinatorial study of convex polytopes and related structures

The lecture will discuss some of the main problems and some of the most disturbing problems in the combinatorics of convex polytopes and more general structures. Among the issues I will discuss are: f-vectors, flag vectors, neighborliness, low-dimensional skeleta, the g-conjecture for spheres, some problems around the Hirsch conjecture, problems on special classes of polytopes.

Eran Nevo
Ben-Gurion University of the Negev

On the generalized lower bound conjecture

The study of face numbers of polytopes is a classical problem. For a simplicial d-polytope P, its face numbers are conveniently encoded by the so called h-numbers $h_0(P), \ldots, h_d(P)$. In 1971, McMullen and Walkup posed the following conjecture, which is called "the generalized lower bound conjecture": (1) If P is a simplicial d-polytope then $h_0(P) \leq h_1(P) \leq \cdots \leq h_{[d/2]}(P)$. (2) Moreover, if $h_{r-1} = h_r$ for some $r \leq d/2$, then P can be triangulated without introducing simplices of dimension $\leq d-r$. Part (1) was proved by Stanley in 1980 using the hard Lefschetz theorem for projective toric varieties. Here we prove part (2), then generalize it to triangulated spheres admitting the weak Lefschetz property. The proof of part (2) uses algebraic, geometric and topological arguments. This is a joint work with Satoshi Murai.

Christos Athanasiadis

University of Athens

Some recent results on subdivisions and local h-vectors

The enumerative theory of subdivisions of simplicial complexes was developed by Stanley in order to understand the effect of simplicial (and more general type of) subdivision on the h-vector of a simplicial complex. A key role in the theory is played by the concept of a local h-vector. This talk will focus on (a) applications of this theory to subdivisions of flag simplicial homology spheres and their γ-vectors, and (b) analogues to cubical subdivisions and cubical h-vectors. Specific examples of simplicial subdivisons, for which the computation of the local h-vector leads to interesting combinatorial problems, will also be discussed.

Benjamin Braun
University of Kentucky

Euler–Mahonian statistics via polyhedral geometry

A variety of descent and major-index statistics have been defined for symmetric groups, hyperoctahedral groups, and their generalizations. Typically associated to pairs of such statistics is an Euler–Mahonian distribution, a bivariate generating function identity encoding these statistics. We use techniques from polyhedral geometry to establish new multivariate generalizations for many of the known Euler–Mahonian distributions. The original bivariate distributions are then straightforward specializations of these multivariate identities. A consequence of these new techniques are bijective proofs of the equivalence of the bivariate distributions for various pairs of statistics. This is joint work with Matthias Beck.

Akimichi Takemura
University of Tokyo

Cones of elementary imsets and supermodular functions: a review and some new results

In this talk we give a review of the method of imsets introduced by Studeny (2005) from a geometric point of view. Elementary imsets span a polyhedral cone and its dual cone is the cone of supermodular functions. We review basic facts on the structure of these cones. Then we derive some new results on the following topics: (i) extreme rays of the cone of standardized supermodular functions, (ii) faces of the cones, (iii) small relations among elementary imsets, (iv) some computational results on Markov basis for the toric ideal defined by elementary imsets. This is a joint work with Takuya Kashimura, Tomonari Sei and Kentaro Tanaka.

Nobuki Takayama

Kobe University

Umbrellas of sets of points and divergent hypergeometric series

The umbrella of a point configuration was introduced by [M.Schultz and U.Walther, Irregularity of hypergeometric systems via slopes along coordinate subspaces, *Duke Math. J.* **142** (2008), 465–509]. This combinatorial object constructed from convex hulls is used to construct integral representations of divergent hypergeometric series. This is a joint work with F. Castro-Jimenez, M. Fernandez-Fernandez and T. Koike.

Carl Lee

University of Kentucky

The cd-index, CD-vector, and h-vector of convex polytopes

We will discuss connections between the **cd**-index for convex polytopes, the CD-vector of Jonathan Fine, and the toric h-vector. In particular, we will discuss formulas for converting from one to another, ways to compute them by sweeping the polytope with a hyperplane, and why the CD-vector is especially nice for simple polytopes.

Margaret Bayer

University of Kansas

Generalizations of cyclic polytopes

We look at several constructions of polytopes, all related to attempts to generalize cyclic polytopes. The complete graph K_n is the 1-skeleton of a cyclic polytope of every dimension from 4 to $n-1$. What other graphs can be realized as 1-skeletons of polytopes of many different dimensions? Cyclic polytopes satisfy Gale's evenness condition. What other (nonsimplicial) polytopes satisfy Gale's evenness condition? Subpolytopes of cyclic polytopes are cyclic polytopes. What other polytopes have many subpolytopes that are cyclic polytopes? The talk will include some answers and, perhaps, some more questions.

Fu Liu

University of California, Davis

Perturbation of transportation polytopes

We introduce a perturbation method that can be used to reduce the problem of finding the multivariate generating function (MGF) of a non-simple cone to computing the MGF of simple cones. We then give a universal perturbation that works for any transportation polytope. We apply this perturbation to the family of central transportation polytopes of order kn x n, and obtain formulas for the MGFs of the feasible cones of vertices of the polytope and the MGF of the polytope. The formulas we obtain are enumerated by combinatorial objects. Yemelichev and Kravtsov gave criteria for a transportation polytope to have the maximum number of vertices. Applying our perturbation to central transportation polytopes, we obtain families of polytopes satisfying this criteria. As a result, we recover the formula for the maximum number of vertices of transportation polytopes of order $kn \times n$.

Nan Li
MIT

Combinatorial aspects of the hypersimplex

Given a polytope, we can define its h-vector, where each term?is nonnegative and their sum equals the normalized volume of the polytope. It is well-known that the normalized volume of the hypersimplex is the Eulerian number. Therefore its h-vector provides a refinement of the Eulerian number. We proved a conjecture by Stanley describing this refinement by descents and excedances by a shellable triangulation. We also generalized this result to slices of larger rectangles. As a byproduct, we came up with a new Eulerian statistic, which enjoys some nice equal joint-distributions with some other known permutation statistics. Combinatorial proofs of some of them are still open.

Christian Haase
Goethe-Universität

Some facets of marginal polytopes

A k-way contingency table is an array of $n_1 \times \ldots \times n_k$ non-negative integers $a_{i_1,\ldots,i_k}$. For a subset $\sigma \subset [k]$, the σ-margin is the σ-way contingency table obtained by summing up the $a_{i_1,\ldots,i_k}$ over all i_j with $j \notin \sigma$. Given a simplicial complex Δ on $[k]$, consider the map which assigns to a contingency table the collection of its σ-margins for $\sigma \in \Delta$. Given $n_1, \ldots, n_k$ and Δ, the marginal polytope is the convex hull of the images of the standard unit tables. There is also a different description as representation polytopes of abelian permutation groups. If we take Δ to be two vertices (no edge between them), this polytope is the product of two simplices of dimensions $n_1 - 1$ and $n_2 - 1$, and it corresponds to the matrix defining ordinary 2-way transportation polytopes. In the binary case, i.e., $n_1 = \ldots = n_k = 2$, if Δ is a graph, these polytopes are cut polytopes. Marginal polytopes appear in optimization, in (algebraic) statistics or in tensor decomposition analysis. Still, their facet structure remains mysterious — particularly so in the non-binary case. In joint work with Barbara Baumeister, Benjamin Nill and Andreas Paffenholz, we studied the case where Δ is the triangle graph. We were able to verify that certain known valid inequalities define facets, and to find new classes of previously unknown facet-inequalities. In particular, this yields a class of $0/1$-polytopes of dimension d with polynomially many vertices but exponentially many facets.

Sinai Robins
Nanyang Technological University

Tiling Euclidean space by translations of a polytope, with multiplicity

We survey the problem of covering Euclidean space by possibly overlapping translates of a convex polytope. If we suppose that by translating a convex polytope, almost every point of R^d is covered exactly k times, then we call such a covering of Euclidean space a k-tiling. The investigation of 1-tilings has a long and rich history and began with the work of the crystallographer Fedorov (1885) and continued by the great Mathematician Minkowski (1904). It was later extended by Venkov and McMullen to give a complete characterization of all convex objects that 1-tile Euclidean space. By contrast, for k larger than 1, the collection of polytopes that k-tile is much wider than the collection of polytopes that 1-tile, and there is currently no known analogous characterization for the polytopes that k-tile. We will see some examples. It turns out that if P k-tiles R^d by translations, then it must be centrally symmetric, and its facets must also centrally symmetric. These are the analogues of Minkowski's classical conditions for tiling. This is joint work with Nick Gravin and Dima Shiryaev.

Richard Stanley

MIT

The visibility arrangement and line shelling arrangement of a convex polytope

Let $\mathscr{P}$ be a convex polytope, and let x be a point in the interior of $\mathscr{P}$. We define hyperplane arrangements $V(\mathscr{P})$ and $L(\mathscr{P},x)$, called the *visibility arrangement* and *line shelling arrangement* of $\mathscr{P}$. The regions of $V(\mathscr{P})$ correspond to sets of facets of $\mathscr{P}$ visible from some point. The regions of $L(\mathscr{P},x)$ correspond to line shellings of $\mathscr{P}$ from the point x. If x is "generic," then the matroid defined by $L(\mathscr{P},x)$ is the Dilworth truncation of that matroid defined by $V(\mathscr{P})$. We discuss some special cases, some further aspects, and some generalizations of these observations.

Yvonne Kemper

University of California, Davis

Flows on Simplicial Complexes

Given a graph G, the number of nowhere-zero Z_q-flows $f_G(q)$ is known to be a polynomial in q. In this talk, we extend the definition of nowhere-zero Z_q-flows to simplicial complexes D of dimension greater than one, and prove the polynomiality of the corresponding function $f_D(q)$ for certain q and certain subclasses of simplicial complexes.

Jing Xi

University of Kentucky

The characteristic imset polytope for diagnosis models

In 2010, M. Studený, R. Hemmecke and Linder explored a new algebraic description of graphical models, characteristic imsets. Compare with standard imsets, characteristic imsets have several advantages: they are still unique vector representative of conditional independence structures, they are 0-1 vectors, and they are more intuitive in terms of graphs than standard imsets. After defining characteristic imset polytope as the convex hull of all characteristic imsets for a given set of nodes, they also showed that a model selection in graphical models, which essentially is a problem of maximizing a quality criterion, can be converted into an integer programming problem on the characteristic imset polytope. However, this integer programming problem is very hard in general. Therefore, here we focus on diagnosis models which can be described by Bipartite graphs with a set of m nodes and a set of n nodes for any $m,n \in \mathbb{Z}_+$, and their characteristic imset polytope. In this talk, first, we will show that the characteristic imsets for diagnosis models have very nice properties including that the number of non-zero coordinates is at most is $n \cdot (2^m - 1)$, and with these properties we are able to find a combinatorial description of all edges of the characteristic imset polytopes for diagnosis models. Then we prove that these characteristic imset polytopes are direct products of n many $(2^m - 1)$ dimensional simplicies. Finally, we end this talk with further questions in this topic.

Laura Escobar
Cornell University

Star^1-convex functions on tropical linear spaces of complete graphs

Given a fan Δ and a cone $\sigma \in \Delta$, let $\text{star}^1(\sigma)$ be the set of cones that contain σ and are one dimension bigger than σ. We study two cones of piecewise linear functions defined on Δ: the cone of functions which are convex on $\text{star}^1(\sigma)$ for all cones, and the cone of functions which are convex on $\text{star}^1(\sigma)$ for all cones of codimension 1. We give nice combinatorial descriptions for these two cones given two different fan structures on the tropical linear space of complete graphs. This problem is motivated by an attempt of Gibney and Maclagan to give a polyhedral description of the net cone of divisors of the moduli space $\overline{M}_{0,n}$ using tropical geometry.

Francisco Santos
Universidad de Cantabria

Unimodular triangulations of dilated 3-polytopes

A seminal result in the theory of toric varieties, due to Knudsen and Mumford (1973) asserts that for every lattice polytope P there is a positive integer k such that the dilated polytope kP has a unimodular triangulation. In dimension 3, Kantor and Sarkaria (2003) have shown that $k = 4$ works for every polytope. But this does not imply that every $k \geq 4$ works as well. In this talk we study the values of k for which the result holds showing that:

(1) It contains all composite numbers.
(2) It is an additive semigroup.

These two properties imply that the only values of k that may not work (besides 1 and 2 which are known not to work) are $k \in \{3, 5, 7, 11\}$. An ad-hoc construction also shows that $k = 7$ and $k = 11$ work, except in this case the triangulation cannot be guaranteed to be "standard" in the boundary, so the only open cases are $k = 3$ and $k = 5$.

Alexander M Kasprzyk

Imperial College London

Fano polytopes, mirror symmetry, and mutations

One approach to classifying Fano manifolds is via the study of Fano polytopes and Mirror Symmetry. In the best spirit of the dictionary translating between combinatorics and toric geometry, abstract questions about Fano manifolds become concrete problems concerning Fano polytopes. Furthermore, some natural algebraic operations translate into surprising combinatorial properties. One such example is a ”mutation”, which sheds some light on the interesting property of Ehrhart quasi-period collapse. I hope to illustrate the calculations involved, and explain some of the more interesting behaviour that arrises.

Benjamin Nill

Case Western Reserve University

Ramifications of the degree of lattice polytopes

The degree of a lattice polytope is an invariant originated in Ehrhart theory. It is a useful measure of complexity for lattice polytopes without interior lattice points. In this talk, I will report on some of the results obtained, their relations to toric geometry, and some new ramifications. Recently, in joint work with Arnau Padrol we got interested in a generalization for combinatorial types of polytopes. Here, the combinatorial degree measures ‘almost-neighborliness’. Most of the existing results for lattice polytopes should have analogues in this purely combinatorial setting. I also hope to describe joint work with Benjamin Lorenz on smooth Gorenstein polytopes with fixed Calabi–Yau dimension. This is another algebro-geometric invariant closely related to the degree.

Matthias Köppe
University of California, Davis

Intermediate sums on polyhedra: Ehrhart theory and an application in mixed integer optimization

We study intermediate sums, interpolating between integrals and discrete sums, which were introduced by [A. Barvinok, Computing the Ehrhart quasi-polynomial of a rational simplex, Math. Comp. 75 (2006), 1449–1466]. For a given polytope P with facets parallel to rational hyperplanes and a rational subspace L, we integrate a given polynomial function h over all lattice slices of the polytope P parallel to the subspace L and sum up the integrals. We first develop an algorithmic theory of parametric intermediate generating functions. Then we study the Ehrhart theory of these intermediate sums, that is, the dependence of the result as a function of a dilation of the polytope. We provide an algorithm to compute the resulting Ehrhart quasi-polynomials in the form of explicit step polynomials. These formulas are naturally valid for real (not just integer) dilations and thus provide a direct approach to real Ehrhart theory. The algorithms are polynomial time in fixed dimension. Following A. Barvinok (2006), the intermediate sums also provide an efficient algorithm to compute, for a fixed number k, the highest k Ehrhart coefficients in polynomial time if P is a simplex of varying dimension. We also present an application in optimization, a new fully polynomial-time approximation scheme for the problem of optimizing non-convex polynomial functions over the mixed-integer points of a polytope of fixed dimension, which improves upon earlier work that was based on discretization [J. A. De Loera, R. Hemmecke, M. Köpe and R. Weismantel, FPTAS for optimizing polynomials over the mixed-integer points of polytopes in fixed dimension, *Math. Prog. Ser. A* **118** (2008), 273–290]. The algorithm also extends to a class of problems in varying dimension. The talk is based on joint papers with Velleda Baldoni, Nicole Berline, Jesus De Loera and Michele Vergne.

Greta Panova

University of California, Los Angeles

Counting with polytopes — tableaux enumeration

In this talk we will show how one can use polytopes and their volumes to enumerate objects like linear extensions of posets and in particular standard tableaux. We will apply the approach to derive some easy classical results, but our ultimate goal will be to find a product formula for a new class of tableaux – truncated tableaux of straight and shifted shapes. Besides polytopes we will also use the theory of symmetric functions and pass through several steps of interpretation to arrive at the final answer.

Hidefumi Ohsugi
Rikkyo University

Centrally symmetric configurations of integer matrices

The concept of centrally symmetric configurations of integer matrices is introduced. We study the problem when the toric ring of a centrally symmetric configuration is normal as well as is Gorenstein. In addition, Gröbner bases of toric ideals of centrally symmetric configurations will be discussed. Special attentions will be given to centrally symmetric configurations of unimodular matrices and those of incidence matrices of finite graphs.

Günter M. Ziegler

Freie Universität Berlin

On projectively unique polytopes

Shephard and McMullen had conjectured that there are only finitely many projectively unique polytopes in each fixed dimension. (Moreover, Shephard has assembled a list of 11 projectively unique 4-polytopes that was conjectured to be complete.) This is consistent with the observation by Steinitz that for 2- and 3-dimensional polytopes, the dimension of the realization space grows linearly with the number of vertices. In this lecture, I will present evidence against the Shephard–McMullen conjectures. (Work in progress with Karim Adiprasito.)

Sam Payne

Yale University

Totally mixed faces of Minkowski sums

A face of a Minkowski sum of polytopes is totally mixed if it is the sum of a positive dimensional face from each summand, and the geometric realization of the poset of totally mixed faces appears naturally in the study of tropical complete intersections. I will discuss a few topological properties of this complex of totally mixed faces that can be proved using general results in tropical geometry and Hodge theory, along with related open problems.

Christian Stump

University of Hannover

A new approach to generalized associahedra using brick polytopes

I will introduce a new class of polytopes called brick polytopes associated to subword complexes for finite Coxeter groups. These polytopes turn out to realize an important class of subword complexes including all cluster complexes of finite types. I will moreover present many interesting combinatorial properties of brick polytopes that reduce in the case of cluster complexes to known and unknown combinatorial properties of generalized associahedra. For example, this new approach yields the vertex description of generalized associahedra, and a Minkowski sum decomposition into Coxeter matroid polytopes. This is joint work with Vincent Pilaud.